Facultad de Ciencias Económicas y Empresariales
Universidad de Navarra

Working Paper nº 2/07

Tourism in the Canary Islands: Forecasting using several seasonal time series models

Juncal Cuñado
Universidad de Navarra

Luis A. Gil-Alaña
Universidad de Navarra

Tourism in the Canary Islands: Forecasting using several seasonal time series models
Juncal Cuñado, Luis A. Gil-Alaña
Working Paper No.02/07
February 2007

ABSTRACT

This paper deals with the analysis of the number of tourists travelling to the Canary Islands by means of using different seasonal statistical models. Deterministic and stochastic seasonality is considered. For the latter case, we employ seasonal unit roots and seasonally fractionally integrated models. As a final approach, we also employ a model with possibly different orders of integration at zero and the seasonal frequencies. All these models are compared in terms of their forecasting ability in an out-of-sample experiment. The results in the paper show that a simple deterministic model with seasonal dummy variables and AR(1) disturbances produce better results than other approaches based on seasonal fractional and integer differentiation over short horizons. However, increasing the time horizon, the results cannot distinguish between the model based on seasonal dummies and another using fractional integration at zero and the seasonal frequencies.

Juncal Cuñado
Universidad de Navarra
Depto. Métodos Cuantitativos
Campus Universitario
31080 Pamplona
jcunado@unav.es

Luis A. Gil-Alaña
Universidad de Navarra
Depto. Métodos Cuantitativos
Campus Universitario
31080 Pamplona
alana@unav.es

1. Introduction

Canary Islands was the leading Spanish regional tourist destination during the last winter season representing 33.6% of total tourism (see Ministerio de Industria, Turismo y Comercio, 2006). In addition, the tourism sector represented 32.6% of the GDP in 2003 and provided 67% of the total services product. Furthermore, this activity generated 37.3% of the total employment in that year (Gobierno de Canarias - Consejería de Turismo, 2004), which indicates the importance of modelling tourism for the Canary Islands.[1]

In the literature of tourism modelling and forecasting, we find two main approaches, based on regression models (see for example Melenberg and van Soest, 1996; Kulendran and King, 1997), while others use pure time series analysis (e.g. Kim 1999, Lim and McAleer, 2002; Goh and Law, 2002; Gustavsson and Nordström, 2001; and Brännäs et al., 2002 among others).

A characteristic commonly observed in many time series related with tourism is the seasonal pattern. However, there is little consensus on how seasonality should be treated in empirical applications on aggregate data. Since the statistical properties of different seasonal models are distinct, the imposition of one kind when another is present can result in serious bias or loss of information, and it is therefore useful to establish what kind of seasonality is present in the data.

In this paper, we examine the number of tourist arrivals in the Canary Islands using monthly data, for the time period 1992:01 to 2005:12, which generates 168 observations, and leave the last 24 (2003:01 - 2005:12) for forecasting purposes. The plan of the paper is as follows. In Section 2, we briefly examine the different ways of

[1] Some papers analyzing the tourism in the Canary Islands are Moreno (2003), Hernández (2004), Hernández-López (2004), Díaz-Pérez et al. (2005), Garín-Muñoz (2006) and Hoti et al. (2006) among others.

modelling seasonality in raw time series. Section 3 presents a variety of model specifications to describe the seasonal behaviour of tourism data in the Canary Islands. In Section 4 we choose a single model for each specification, while Section 5 presents a foresting exercise. Finally, Section 6 contains some concluding comments.

2. Seasonal models

Modelling the seasonal component in time series is a matter that still remains controversial. Seasonal dummy variables have been employed for many years but this type of (deterministic) seasonality has been found to be inappropriate in many cases, especially if the seasonal component changes or evolves over time. On the other hand, stochastic stationary seasonal models are usually based on seasonal AutoRegressive-Moving Averages (ARMA) models of form:

$$\phi(L^S)\, y_t \;=\; \theta(L^S)\, \varepsilon_t\,, \quad t = 1, 2, ..., \tag{1}$$

where y_t is the time series we observe, $\phi(L^S)$ is the seasonal AR polynomial, where L^s is the seasonal lag-operator ($L^s y_t = y_{t-s}$) and s is the number of time periods within a year; $\theta(L^S)$ is the MA polynomial and e_t is white noise. In this approach, the roots of the seasonal AR polynomial must be outside the unit circle. If they are in the unit circle, the process contains seasonal unit roots, and seasonal first differences are required to render the series stationary. In other words,

$$(1 - L^S)\, y_t \;=\; u_t\,, \quad t = 1, 2, ..., \tag{2}$$

where u_t is I(0), (defined as a covariance stationary process with spectral density function that is positive and finite at any frequency) and thus, it can be specified in terms of a white noise or any type of weakly (seasonal/non-seasonal) autocorrelated processes. However, the seasonal unit root model described in (2) is merely one of the many mathematical models that may be employed to describe the seasonal structure. In fact, equation (2) can be extended to the case of:

$$(1 - L^s)^d \, y_t \;=\; u_t, \quad t = 1, 2, ..., \tag{3}$$

where d can be any real value. In that case the process is said to be seasonally fractionally integrated, and the polynomial in the left-hand-side of (3) can be expressed in terms of its Binomial expansion such that, for all real d,

$$(1 - L^s)^d \;=\; \sum_{j=0}^{\infty} \frac{\Gamma(d+1)\,(-L^s)^j}{\Gamma(d-j+1)\,\Gamma(j+1)} \;=\; 1 \,-\, d\,L^s \;+\; \frac{d\,(d-1)}{2} L^{2s} \;-\; ...$$

Processes like (3), (with d > 0) belong to the class of seasonal long memory processes, so-named because of the strong association (in the seasonal structure) between observations widely separated in time. The notion of fractional integration with seasonality was suggested by Jonas (1981), and extended in a Bayesian framework by Carlin et al. (1985) and Carlin and Dempster (1989). Porter-Hudak (1990) used a fractional model like (3) (with s = 4) to some quarterly US monetary aggregates, and other empirical works in this context are Ray (1993), Sutcliffe (1994) and more recently, Gil-Alana and Robinson (2001) and Gil-Alana (2002).

On the other hand, most of the literature on fractional integration has concentrated on the long run or zero frequency (i.e., using a polynomial of form: $(1\text{-}L)^d$ rather than $(1\text{-}L^s)^d$), and it has been identified for several macroeconomic time series in many papers. This finding is often explained using Robinson (1978) and Granger (1980) aggregation results: cross section aggregation of a large number of AR(1) processes with heterogeneous AR coefficients may create long memory at the zero frequency. Parke (1999) uses a closely related discrete time error duration model, while Diebold and Inoue (2001) relate fractional integration with regime switching models. Lildholdt

(2002) provides both theoretical and Monte Carlo evidence that the three types of explanations may also generate seasonal fractional integration of form as in (3).[2]

In this paper we examine the number of monthly arrivals in the Canary Islands using different models for the seasonal structure investigating which is the best approach in terms of their forecasting properties.

3. Empirical analysis

The data analysed in the paper refer to the number of tourist arrivals in the Canary Islands for the time period 1992:01 to 2005:12. The data are monthly, seasonally unadjusted, and are obtained from the National Airport Administration (AENA) at airports from information regarding the number of tourist arrivals.[3] In Figure 1 we present the plots of the original series, its first seasonal differences, and the correlograms and periodograms of these two series.

[Insert Figure 1 about here]

Starting with the original series we observe a clear seasonal pattern and this is substantained by both the correlogram and the periodogram. Looking at the seasonally differenced series, seasonality seems to be removed, though the correlogram still presents some significant values at some lags suggesting that other degrees of differentiation may be more appropriate than first differences.

In order to model and forecast the series, first we assume that the seasonal component is deterministic, and consider the model,

$$y_t = \gamma + \beta t + \sum_{j=1}^{s-1} \gamma_j S_{jt} + u_t, \quad t = 1, 2, \ldots, \qquad (4)$$

---

[2] Lildholdt (2002) shows that fractional integration at the seasonal frequencies may be created by: a) cross-sectional aggregation of seasonal data; b) aggregation of seasonal duration models, and c) regime-switching if the underlying Markov process possesses seasonal dependencies.

[3] Hoti et al. (2006) and Garin-Munoz (2006) also use the same data base.

with s = 12, S_{it} refers to the seasonal (monthly) dummy variables, and u_t is modelled as a white noise process but also with autocorrelation through AR processes. We estimate the model including only an intercept and with an intercept and a linear time trend. The results are presented in Table 1.

[Insert Table 1 about here]

We estimate model (4), study the significance of the coefficients, and perform a global significance test for the null hypothesis of no seasonality (all the γ_i-coefficients equal to zero). As shown in Table 1, we can reject the null hypothesis of no seasonality in all the four estimated models. We also present various selection criteria (AIC, SIC) in order to select which of these models is preferable to explain the behaviour of the series. Based on these criteria, we conclude that the model with an intercept and a linear time trend and AR(1) u_t is the preferable one.

[Insert Figure 2 about here]

Figure 2 displays the time series evolution of the series for each month. A slight changing pattern is observed in all cases, with the values increasing smoothly across time, implying that the seasonal component of the series may be nontationary. Thus, as a second approach, we consider the case of seasonal first differences and perform various methods for testing such hypothesis. In particular, we use the procedure developed by Dickey, Hasza and Fuller (DHF, 1984) and the extension of the tests of Hylleberg, Engle, Granger and Yoo (HEGY, 1990) to the monthly case (Beaulieu and Miron, 1993; Franses, 1991).[4]

The DHF test is basically an extension of the tests of Dickey and Fuller (1979) to processes such as:

$$(1 - \rho_S L^S)\, y_t \; = \; \varepsilon_t \qquad (5)$$

where ?$_s$ = 1. The test is based on the auxiliary regression of form:

$$(1 - L^s) y_t \; = \; \pi y_{t-s} \; + \; \varepsilon_t \qquad (6)$$

an the test statistic is the t-value corresponding to p in (6). Due to the nonstandard asymptotic distributional properties of the t-values under the null, H_o: p = 0, DHF (1984) provide the fractiles of simulated distributions which give us the critical values to be applied when testing the null against the alternative H_1: p < 0. In order to whiten the errors in (6), the auxiliary regression may be augmented by lagged values of $(1 - L^s)$ y_t, and with deterministic parts as an intercept or a linear trend, but unfortunately this changes the distribution of the test statistic.[5]

[Insert Table 2 about here]

Table 2 displays the results based on the above approach. We observe that in all except one case (white noise u_t with an intercept) we find evidence of seasonal unit roots. Using the tests developed by Beaulieu and Miron (1993) the conclusions were practically the same as with the DHF test, and we found evidence of unit roots in practically all cases.[6]

As a third alternative approach we suppose that the seasonal component may be fractionally integrated and consider models of form as in (3). Here, we employ a procedure suggested by Robinson (1994). (See, Gil-Alana, 2002 for an application of this approach). His method consists of testing the null hypothesis:

$$H_o : \; d \; = \; d_o , \qquad (7)$$

for any real value d_o, in a model given by:

$$y_t \; = \; b' z_t \; + \; x_t , \qquad t \; = \; 1, 2, \ldots, \qquad (8)$$

---

[4] Other seasonal unit root tests are Ghysels et al. (1994), Canova and Hansen (1995) and Tam and Reimsel (1997), the latter proposing a test for a unit root in the seasonal MA operator.

[5] The critical values are tabulated in Franses and Hobijn (1997).

and x_t given by (3), i.e.,

$$(1 - L^{12})^d\, x_t \;=\; u_t\,, \;\; t = 1, 2, ..., \qquad (9)$$

where y_t is the observed time series; $\beta = (\beta_1, \ldots, \beta_k)^T$ is a (kx1) vector of unknown parameters; and z_t is a (kx1) vector of deterministic regressors that may include, for example, an intercept, (e.g., $z_t \equiv 1$), or an intercept and a linear time trend, (in case of $z_t = (1,t)^T$). The functional form of the test statistic, (denoted by $\hat{r}$), is described in Appendix A.

Based on H_o (7), Robinson (1994) showed that under certain very mild regularity conditions,

$$\hat{r} \;\;\rightarrow_d\;\; N(0,1) \quad as \quad T \rightarrow \infty. \qquad (10)$$

Thus, an approximate 100α% level test of (7) will reject H_o against the alternative: H_a: $d > d_o$ ($d < d_o$) if $\hat{r} > z_\alpha$ ($\hat{r} < -z_\alpha$), where the probability that a standard normal variate exceeds z_α is α. He also showed the efficiency property of the test against local departures from the null.

[Insert Table 3 about here]

The results displayed in Table 3 refer to the 95% confidence intervals of those values of d_o where H_o cannot be rejected for the three cases of no regressors, an intercept and an intercept with a linear trend. We also included seasonal dummies in the deterministic component z_t in (8) but they were found to be statistically insignificant in practically all cases.[7] We also display in the tables (in parenthesis within the brackets)

---

[6] An advantage of the Beaulieu and Miron's (1993) approach is that it permits us to test for seasonal unit roots without maintaining that roots are present at all the frequencies. However, using this method, we cannot reject that unit roots are present at all frequencies at the 5% significance level.

[7] This may be a consequence of the interaction with the seasonal fractional polynomial given by equation (9). Note that the model $y_t = \alpha + \sum_{j=1}^{11} \gamma_j S_{jt} + x_t$; $(1 - L^{12})^d x_t = u_t$ can also be expressed as $(1 - L^{12})^d y_t = \alpha (1 - L^{12})^d 1_t + \sum_{j=1}^{11} \gamma_j w_{jt} + u_t$, where $w_{jt} = (1 - L^{12})^d S_{jt}$.

the value of d_o producing the lowest statistic in absolute value. That value should be an approximation to the maximum likelihood estimate.[8] As in the previous cases we assume that u_t is white noise and AR(1). We see that the intervals are relatively large, which may be a consequence of the small sample size used in this application. If u_t is white noise the values are all strictly above 0 and the unit root null hypotheses cannot be rejected, which is consistent with the results above based on seasonal unit roots. Moreover, the estimates substantially change depending on the inclusion of no regressors, an intercept, and an intercept and a linear trend. If u_t is modelled in terms of an AR(1) process, the lowest statistics take place at $d_o = 0.14$ (no regressors); 0.18 (with an intercept) and 0.22 (with a linear trend).

As an additional method, we also employ a simpler version of the tests of Robinson (1994) where only the root at the zero frequency is taken into account. In other words, we test the same model as before, i.e. the one given by equation (8), with

$$(1 - L)^d \, y_t \; = \; u_t, \quad t = 1, 2, ..., \tag{11}$$

assuming that the disturbances are white noise and a seasonal AR(1) process of form:

$$u_t \; = \; \rho u_{t-12} + \varepsilon_t, \quad t = 1, 2, ... \tag{12}$$

Then, the seasonal component is supposed to be stationary, and the nonstationarity comes from the (fractionally-) differenced polynomial in (11) and/or the linear trend. Here, the test statistic takes the same form as $\hat{r}$ in the Appendix, with the only difference that $y(l_j) = \log\left|2\sin\frac{l_j}{2}\right|$, and $\hat{u}_t = (1-L)^{d_o} y_t - \hat{b}\,(1-L)^{d_o} z_t$. The results based on this approach are displayed in Table 4.

[Insert Table 4 about here]

8 Note that this method is based on the Whittle function, which is an approximation to the likelihood function.

We observe that if u_t is white noise, the values of d range widely between 0.01 (with a linear time trend) and 0.79 (no regressors). However, if u_t is seasonal AR, the values are higher, and the lowest statistics are obtained at d = 0.33 (with no regressors); 0.43 (with an intercept), and 0.56 (with a linear trend).

The last two approaches to investigating the seasonal behaviour of the series consist in testing parametric models for the series of interest, relying on the seasonal implications of the estimated models. The advantage of this procedure is the precision gained by providing all the information about the series through the parameter estimates. A drawback is that these estimates are sensitive to the class of models considered, and may be misleading because of misspecification. It is well known that the issue of misspecification can never be settled conclusively in the case of parametric (or even semiparametric) models. However, the problem can be partly addressed by considering a larger class of models. Note that the model used in (9) is based on the polynomial $(1 - L^{12})^d$, which may be decomposed into $(1 - L)^d$ and $S(L)^d$ where $S(L) = (1 + L + \ldots + L^{11})$. That implies that we are imposing the same degree of integration, d, at the zero frequency ((1-L)) and at the seasonal ones (S(L)). In what follows, we employ another version of the tests of Robinson (1994) that enables us to simultaneously consider roots at zero and the seasonal frequencies.

For this purpose, let us consider again the model given by (8), with

$$(1 - L)^{d_1} \; (1 - L^{12})^{d_2} \; x_t \; = \; u_t, \quad t \; = \; 1, 2, \ldots, \tag{13}$$

with $z_t = (1,t)^T$. Thus, under the null hypotesis:

$$H_o : d \equiv (d_1, d_2) \; = \; (d_{1o}, d_{2o}) \equiv \; d_o, \tag{14}$$

the model becomes:

$$y_t \; = \; \beta_0 \; + \; \beta_1 t \; + \; x_t, \quad t \; = 1, 2, \ldots \tag{15}$$

$$(1 - L)^{d_{1o}} \; (1 - L^{12})^{d_{2o}} \; x_t \; = \; u_t, \quad t \; = \; 1, 2, \ldots, \tag{16}$$

where d_{1o} and d_{2o} are real values, and, if $d_{1o} = 0$, the model reduces to the case previously studied in Table 3, and if $d_{2o} = 0$, the one presented in Table 4. As before, we examine separately the cases of $\beta_0 = \beta_1 = 0$ a priori (i.e., with no regressors in the undifferenced model (15)); β_0 unknown and $\beta_1 = 0$ (with an intercept); and β_0 and β_1 unknown (with an intercept and a linear time trend). The results of the estimated values of d_{1o} and d_{2o} with their corresponding intervals are displayed in Table 5.

[Insert Table 5 about here]

We observe in Table 5 that if we do not include regressors or only an intercept is included, the values of d_{1o} and d_{2o} are very similar in the two cases independently of the use of white noise or seasonal AR(1) u_t.[9] It is around 0.40 for the long run or zero frequency, and slightly above 0 for the seasonal polynomial $(1-L^{12})$. Including a linear time trend, the orders of integration are 0.13 and 0.02 in case of white noise disturbances, and 0.14 and 0.04 with AR(1) u_t.

4. Model selection

In Section 3 we have presented a variety of model specifications to describe the seasonal structure of the series. In this section we choose a single model for each specification according to various criteria. Starting with the deterministic approach, the results in Table 1 suggests that the best model specification is:

$$y_t = 1605913 + 6861.3t + 67115.8S_{1t} - 22939.1S_{2t} + 267538.7S_{3t} + 134419.5S_{4t} - 279691.7S_{5t} - 291705.6S_{6t} + 86030.5S_{7t} + 312894.3S_{8t} + 16125.7S_{9t} + 118581.1S_{10t} + 67887.9S_{11t} + u_t,$$

$$u_t = 0.661u_{t-1} + \varepsilon_t. \qquad \text{(DUM)}$$

However, permitting a seasonal unit root, (Table 2) the best specification according to the results based on Dickey, Hasza and Fuller’s (1984) tests seems to be:

$$(1-L^{12})y_t = 230022.5 - 0.068361 y_{t-12} + u_t \qquad u_t = 0.536 u_{t-1} + \varepsilon_t .^{10} \qquad \text{(DHF)}$$

The third approach uses seasonal fractional integration (Table 3), and based once more on the significance of the coefficients, the selected model is:

$$y_t = 2171261.72 + x_t; \qquad (1 - L^{12})^{0.17} x_t = u_t; \qquad \text{(SFI)}$$
$$u_t = 0.808 u_{t-1} + e_t .$$

Similarly, using fractional integration exclusively at the long run or zero frequency, the selected model is:

$$y_t = 1825827.57 + x_t; \qquad (1 - L)^{0.56} x_t = u_t; \qquad \text{(FI)}$$
$$u_t = 0.894 u_{t-12} + \varepsilon_t .$$

The final approach, based on the two fractional polynomials at zero and the seasonal frequencies leads to the model:

$$y_t = 2171514.89 + x_t;$$
$$(1 - L)^{0.38} (1 - L^{12})^{0.06} x_t = u_t; \qquad u_t = 0.820 u_{t-1} + \varepsilon_t . \qquad \text{(FSFI)}$$

In the following section we examine the forecasting ability of the prescribed models based on the last 24 out-of-sample observations. We denote each of the models as follows: DUM refers to the model based on seasonal dummy variables; DHF, the one obtained through the Dickey, Hasza and Fuller's (1982) procedure and based on seasonal unit roots; SFI, the one based on seasonal fractional integration; FI, the one using fractional integration at the zero frequency; and finally, FSFI, the combination of the last two fractional processes, i.e., with roots at zero and the seasonal frequencies.

5. A forecasting exercise

In this section we compare the models presented in Section 4 in terms of their forecasting performance. The accuracy of different forecasting methods is a topic of

[9] Non-seasonal AR(1) u_t were also tried here, and the results were fairly similar to those based on white noise u_t.

continuing interest and research.[11] Standard measures of forecast accuracy are the following: the mean absolute percentage error (MAPE), the mean-squared error (MSE), the root-mean-squared error (RMSE), the root-mean-percentage-squared error (RMPSE) and mean absolute deviation (MAD) (Witt and Witt, 1992). Let y_t be the actual value in period t; f_t the forecast value in period t, and n the number of periods used in the calculation. Then:

a) Mean absolute percentage error (MAPE): $\frac{\sum |(y_t - f_t)/y_t|}{n}$;

b) Mean squared error (MSE): $\frac{\sum (y_t - f_t)^2}{n}$;

c) Root-mean-percentage-squared error (RMPSE): $\sqrt{\frac{\sum (y_t - f_t)^2 / f_t}{n}}$;

d) Root-mean-squared error (RMSE): $\sqrt{\frac{\sum (y_t - f_t)^2}{n}}$;

e) Mean absolute deviation (MAD): $\frac{\sum |y_t - f_t|}{n}$.

The MAD measures the magnitude of the forecast errors. Its principal advantage is the ease of interpretation though it ignores the importance of over- or under-estimation. The second type of accuracy measure is based on the forecast error, which is the difference between the observation, y_t, and the forecast, f_t. This category includes MSE, RMSE and RMPSE. MSE is simply the average of squared errors for all forecasts. It is suitable when more weight is to be given to big errors, but it has the drawback of being overly sensitive to a single large error. Further, just like MAD, it is not informative about whether a model is over- or under-estimating compared to the true values. RMSE is the square root of MSE and is used to preserve units. RMSPE

[10] This model is based on the lack of significance of the time trend and the non-zero AR(1) coefficient.

differs from RMSE in that it evaluates the magnitude of the error by comparing it with the average size of the variable of interest. The main limitation of all these statistics is that they are absolute measures for a specific series, and hence do not allow comparisons across different time series and for different time intervals. By contrast, this is possible using a third type of accuracy measure, such as MAPE, which is based on the relative or percentage error. This is particularly useful when the units of measurement of y are relatively large. However, MAPE also fails to take over- or under-estimation into consideration.

[Insert Table 6 about here]

The last 24 out-of-sample forecast errors for the five selected models of Section 4 are displayed in Table 6. We observe that in most cases, the model based on dummy variables (DUM) produces the lowest values, followed by the FSFI model, that is, the one with roots at both the zero and the seasonal frequencies. This model produces the lowest forecast errors in 4 out of the 24 values and outperforms the other three based on integer or fractional differentiation (DHF, SFI and FI) over all time horizons in all cases.

[Insert Table 7 about here]

In Table 7 we display the values of the five forecasting criteria described above for the five specifications of the seasonal case. The DUM model produces the lowest values in all cases, again followed by the FSFI model.

The above measures used for comparing the relative forecasting performance of our models are purely descriptive devices. There exist several statistical tests for comparing different forecasting models. One of these tests, widely employed in the time

[11] See Makridakis et al. (1998) for a study on the forecasting accuracy of major forecasting models, and Makridakis and Hibon (2000) for a summary and review of forecasting competition.

series literature, is the asymptotic test for a zero expected loss differential of Diebold and Mariano (1995).[12] The loss differential is defined as

$$d_t = g(e_{it|t-h}) - g(e_{jt|t-h}),$$

where $g(e_{it|t-h})$ is the loss function, and $e_{it|t-h}$ is the corresponding h-step ahead forecast error for the model i, $e_{it|t-h} = y_t - \hat{y}_{it|t-h}$. Given a covariance stationary sample realization $\{d_t\}_{t=T+h,\ldots,T+n}$, the Diebold-Mariano statistic for the null hypothesis of equal forecast accuracy (i.e., $E(d_t = 0)$) is given by: $\frac{\bar{d}}{\sqrt{\hat{V}(\bar{d})}}$, where $\bar{d}$ is the sample mean loss differential, $\bar{d} = \frac{1}{n-h+1}\sum_{t=T+h}^{t=T+n} d_t$, and where $\hat{V}(\bar{d})$ is a consistent estimate of the asymptotic variance of $\bar{d}$, which is computed as an unweighted sum of the sample autocovariances, that is, $\hat{V}(\bar{d}) = \frac{1}{n-h+1}\left(\hat{\gamma}_0 + 2\sum_{k=1}^{h-1}\hat{\gamma}_k\right)$, where $\hat{g}_k = \frac{1}{n-h+1} \sum_{t=T+h+k}^{T+n}(d_t - \bar{d})(d_{t-k} - \bar{d})$. Harvey, Leybourne and Newbold (1997) note that the Diebold-Mariano test statistic could be seriously over-sized as the prediction horizon, h, increases, and therefore provide a modified Diebold-Mariano test statistic given by:

$$\mathrm{M-DM} = \mathrm{DM}\sqrt{\frac{n + 1 - 2h + h(h-1)/n}{n}},$$

where DM is the original Diebold-Mariano statistic. Harvey et al. (1997) and Clark and McCracken (2001) show that this modified test statistic performs better than the DM test statistic in finite samples, and also that the power of the test is improved when p-values are computed with a Student t-distribution.

[Insert Table 8 about here]

---

[12] An alternative approach is the bootstrap-based test of Ashley (1998), though this method is

Using the M-DM test statistic, we further evaluate the relative forecast performance of the different models by making pairwise comparisons. We focus on the DUM and the FSFI models, and use the absolute and squared prediction errors in the computations. The results are displayed in Table 8. We observe that the test statistic rejects the null hypothesis that model DUM and model FSFI’s forecast performances are equal in favour of the one-sided alternative that model DUM's performance is superior at the 5% level for short horizons. However, increasing the time horizon, the results cannot distinguish between one model or the other.

6. Conclusions

In this paper we have analyzed the number of tourist arrivals in the Canary Islands, monthly, (seasonally unadjusted), for the time period 1992:01 - 2005:12. We use different seasonal time series models and look at the forecasting ability of the proposed models. The results show that a simple deterministic model based on seasonal dummy variables with AR(1) disturbances produces the best results over short horizons, outperforming other more complicated approaches based on seasonal fractional/integer differentiation. However, increasing the time horizon, the results cannot distinguish between the model with dummies and another one based on seasonal fractional integration at zero and the seasonal frequencies. In fact, this latter model outperforms those using single fractional polynomials either at zero or at the seasonal structure.

The fact that a simple deterministic model with an AR(1) structure for the disturbance term is the best specification for this series may be explained by the nature of the series of itself. Thus, the nonstationarity is described by a linear time trend, and the significance of the seasonal dummy variables suggests that the monthly structure of

computationally more intensive.

the series has not changed over time, while the AR(1) polynomial describes the short-run dynamics of the series. It is also interesting to note that seasonal differencing, which is a standard practice when dealing with seasonal data, is outperformed by the fractional model, and that a larger model containing fractional orders of integration at zero and the seasonal frequencies outperform other rival structures. Finally, the use of other approaches like time-varying coefficient models (e.g., Osborn and Smith, 1989, etc.) should also be taken into account.

Appendix

The test statistic proposed by Robinson (1994) for testing H_o (7) in (3) and (8) is derived from the Lagrange Multiplier (LM) principle, and takes the form:

$$\hat{r} = \left(\frac{T}{\hat{A}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2},$$

where T is the sample size, and

$$\hat{a} = \frac{-2\pi}{T} \sum_{j}^{*} \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); \quad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j);$$

$$\hat{e}(l_j) = \frac{\partial}{\partial t} \log g(l_j; \hat{t})$$

$$\hat{A} = \frac{2}{T}\left(\sum_{j}^{*} y(l_j) y(l_j)' - \sum_{j}^{*} y(l_j) \hat{e}(l_j)' \left(\sum_{j}^{*} \hat{e}(l_j) \hat{e}(l_j)' \right)^{-1} \sum_{j}^{*} \hat{e}(l_j) y(l_j)' \right)$$

$$y(l_j) = \mathrm{Re}\left(\frac{\partial}{\partial q} \log r(e^{i l_j}; 0) \right) = \log\left| 2 \sin \frac{l_j}{2} \right| + \log\left(2 \cos \frac{l_j}{2} \right) + \log\left| 2 \cos l_j \right| + \log\left| 2\left(\cos l_j - \cos \frac{p}{3} \right) \right|$$
$$+ \log\left| 2\left(\cos l - \cos \frac{2p}{3} \right) \right| + \log\left| 2\left(\cos l - \cos \frac{p}{6} \right) \right| + \log\left| 2\left(\cos l - \cos \frac{5p}{6} \right) \right|$$

$I(\lambda_j)$ is the periodogram of $\hat{u}_t = (1-L^{12})^{d_o} y_t - \hat{b}\,(1-L^{12})^{d_o} z_t$, and $\hat{t} = \arg\min_{t \in T^*} s^2(t)$, with T^* as a suitable subset of the R^q Euclidean space. The sum on * is over $\lambda_j = 2\pi j/T$, such that $-\pi < \lambda_j < \pi$. $\lambda_j \notin (\rho_l - \lambda_1, \rho_l + \lambda_1)$, l = 1, 2, …, s such that ρ_l, l = 1, 2, …, s < ∞ are the distinct poles of $\rho(L)$.

References

Ashley, R., 1998, A new technique for postsample model selection and validation, Journal of Economics Dynamics and Control 22, 647-665.

Beaulieu, J.J. and J.A. Miron, 1993, Seasonal unit roots in aggregate US output, Journal of Econometrics 55, 305-328.

Brännäs, K., Hellström, J. and Nordström, J., 2002, A new approach to modelling and forecasting monthly guest nights in hotels, International Journal of Forecasting 18, 19-30.

Canova, F. and B. S. Hansen, 1995, Are seasonal patterns constant over time? A test for seasonal stability, Journal of Business and Economic Statistics 13, 237-252.

Carlin, J. B. and A. P. Dempster, 1989, Sensitivity analysis of seasonal adjustments: Empirical cases studies, Journal of the American Statistical Association 84, 6-20.

Carlin, J. B., A. P. Dempster and A. B. Jonas, 1985, On methods and moments for Bayesian time series analysis, Journal of Econometrics 30, 67-90.

Clark, T.E. and M.W. McCracken, 2001, Tests of forecast accuracy and encompassing for nested models, Journal of Econometrics 105, 85-110.

Díaz-Pérez, F.M., M. Bethencourt-Cejas and J.A. Alvarez-Gonzalez, 2005, The segmentation of Canary Island tourism markets by expenditure: implications for tourism policy, Tourism Management 26, 961-964.

Dickey, D.A., D.P. Hasza and W.A. Fuller, 1984, Testing for unit roots in seasonal time series, Journal of the American Statistical Association 79, 355-367.

Dickey, D. A. and W. A. Fuller, 1979, Distribution of the estimators for autoregressive time series with a unit root, Journal of the American Statistical Association 74, 427-431.

Diebold, F.X. and A. Inoue, 2001, Long memory and regime switching, Journal of Econometrics 105, 131-159.

Diebold, F.X. and R.S. Mariano, 1995, Comparing predictive accuracy, Journal of Business, Economics and Statistics 13, 253-263.

Franses, P.H., 1991, Seasonality, nonstationarity and forecasting of monthly time series, International Journal of Forecasting 7, 199-208.

Franses, P.H. and B. Hobijn, 1997, Critical values for unit root tests in seasonal time series, Journal of Applied Statistics 24, 25-47.

Garín-Muñoz, T., 2006, Inbound international tourism to Canary Islands: A dynamic panel data model, Tourism Management 27, 281-291.

Ghysels, E., H. S. Lee and J. Noh, 1994, Testing for unit roots in seasonal time series: some theoretical extensions and a Monte Carlo investigation, Journal of Econometrics 62, 415-443.

Gil-Alana, L.A., 2002, Seasonal long memory in the aggregate output, Economics Letters 74, 333-337.

Gil-Alana, L. A. and P. M. Robinson, 2001, Testing of seasonal fractional integration in U.K. and Japanese consumption and income, Journal of Applied Econometrics 16, 95-114.

Gobierno de Canarias - Consejería de Turismo, 2004, Cuenta impactur de Canarias: Avance de resultados, Comunicado de Prensa.

Goh, C. and R. Law, 2002, Modelling and forecasting tourism demand for arrivals with stochastic nonstationary seasonality and intervention, Tourism Management 23, 499-510.

Granger, C.W.J., 1980, Long memory relationships and aggregation of dynamic models, Journal of Econometrics 14, 227-238.

Gustavsson; P. and J. Nordström, 2001, The impact of seasonal unit roots and vector ARMA modelling on forecasting montly tourism flows, Tourism Economics 7, 117-133.

Harvey, D.I., S.J. Leybourne and P. Newbold, 1997, Testing the equality of prediction mean squared errors, International Journal of Forecasting 13, 281-291.

Hernández, R., 2004, Impact of tourism consumption on GDP. The role of imports, Nota Di Lavoro 27.

Hernandez-Lopez, M., 2004, Future tourists' characteristics and decisions: The use of genetic algorithms as a forecasting, Tourism Economics 10, 245-62.

Hoti, S., León, C. and M. McAleer, 2006, International tourism demand and volatility models for the Canary Islands, Mimeo.

Hylleberg, S., R. F. Engle, C. W. J. Granger and B. S. Yoo, 1990, Seasonal integration and cointegration, Journal of Econometrics 44, 215-238.

Jonas, A. B., 1981, Long memory self similar time series models, unpublished manuscript, Harvard University, Department of Statistics.

Kim, J.H., 1999, Forecasting monthly tourists departures from Australia, Tourism Economics 5, 277-291.

Kulendran, N. and M.L. King, 1997, Forecasting international quarterly tourist flows using error-correction and time series models, International Journal of Forecasting 13, 319-327.

Lildholdt, P.M, 2002, Sources of seasonal fractional integration in macroeconomic time series, Centre for Analytical Finance, University of Aarhus, Working Paper, No. 125.

Lim, C. and McAleer. M., 2002, Time series forecasts of international travel demand for Australia, Torism Management 23, 389-396.

Makridakis, S., S. Wheelwright and R. Hyndman, 1998, Forecasting methods and applications, 3rd edition, John Wiley & Sons.

Makridakis, S. and M. Hibon, 2000, The M-3 competition: results, conclusions and implications, International Journal of Forecasting 16, 451-476.

Melenberg, B. and A. van Soest, 1996, Parametric and semi-parametric modelling of vacation expenditures, Journal of Applied Econometrics 11, 59-76

Ministerio de Industria, Turismo y Comercio, 2006, Diez indicadores mensuales de la economía española, Subdirección General de Estudios y Planes de Actuación.

Moreno, S., 2003, Tourism development in the Canary, Annals of Tourism Research 30, 744-747.

Osborn, D.R. and J.P. Smith, 1989, The performance of period AR models in forecasting seasonal UK consumption, Journal of Business and Economic Statistics 7, 117-127.

Parke, W.R., 1999, What is fractional integration? The Review of Economics and Statistics, 81, 632-638.

Porter-Hudak, S., 1990, An application of the seasonal fractionally differenced model to the monetary aggregate, Journal of the American Statistical Association 85, 338-344.

Ray, B. K., 1993, Long range forecasting of IBM product revenues using a seasonal fractionally differenced ARMA model, International Journal of Forecasting 9, 255-269.

Robinson, P.M., 1978, Statistical inference for a random coefficient autoregressive model, Scandinavian Journal of Statistics 5, 163-168.

Robinson, P.M., 1994, Efficient tests of nonstationary hypotheses, Journal of the American Statistical Association 89, 1420-1437.

Sutcliffe, A., 1994, Time series forecasting using fractional differencing, Journal of Forecasting 13, 383-393.

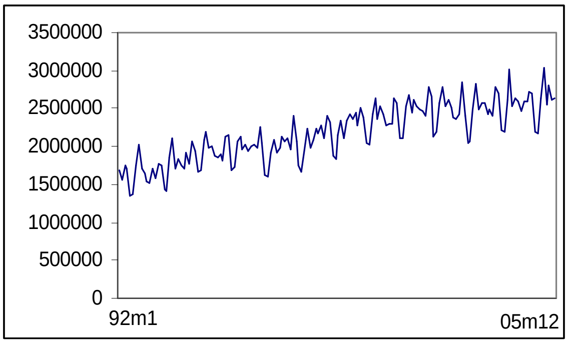
Tam, W. and G. C. Reimsel, 1997, Tests for seasonal moving average unit root in ARIMA models, Journal of the American Statistical Association 92, 725-738.

Witt, S.F. and C.A. Witt, 1992, Modelling and forecasting demand in tourism, London Academic Press, London, UK.

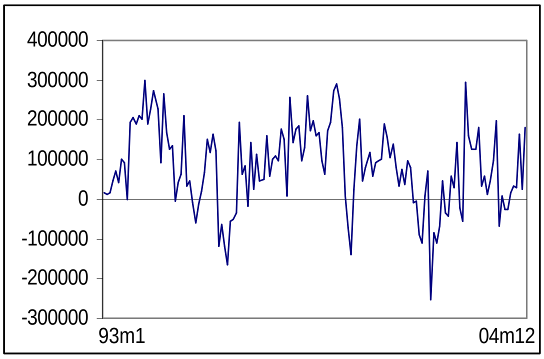
FIGURE 1

Original series and first seasonal differences with their corresponding correlograms and periodograms

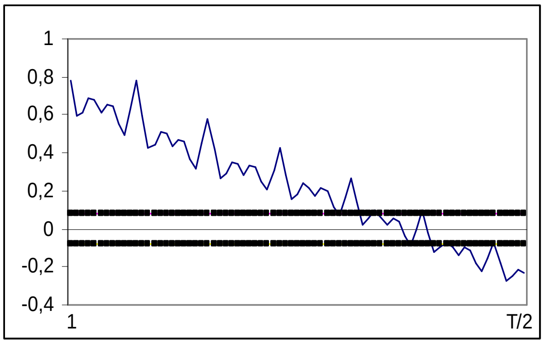
Original series



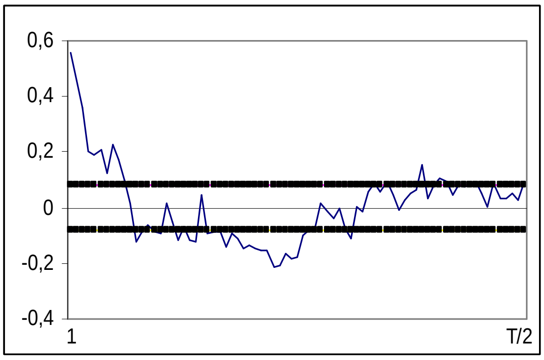
First seasonal differences



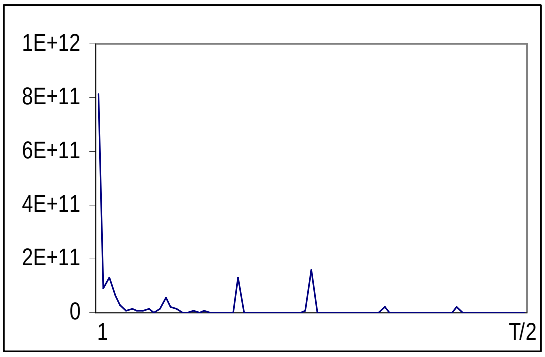
Correlogram original series



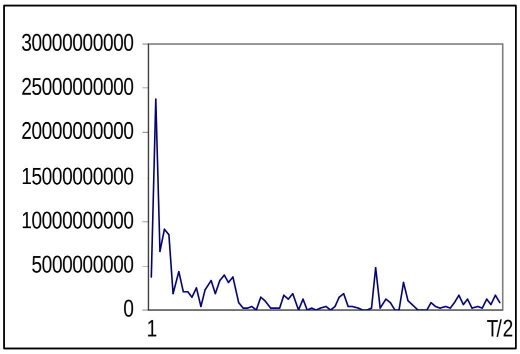
Correlogram first seasonal differences



Periodogram original series



Periodogram first seasonal differences



The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.077 for series of length considered here.

FIGURE 2

Time series evolution for each of the months in the year

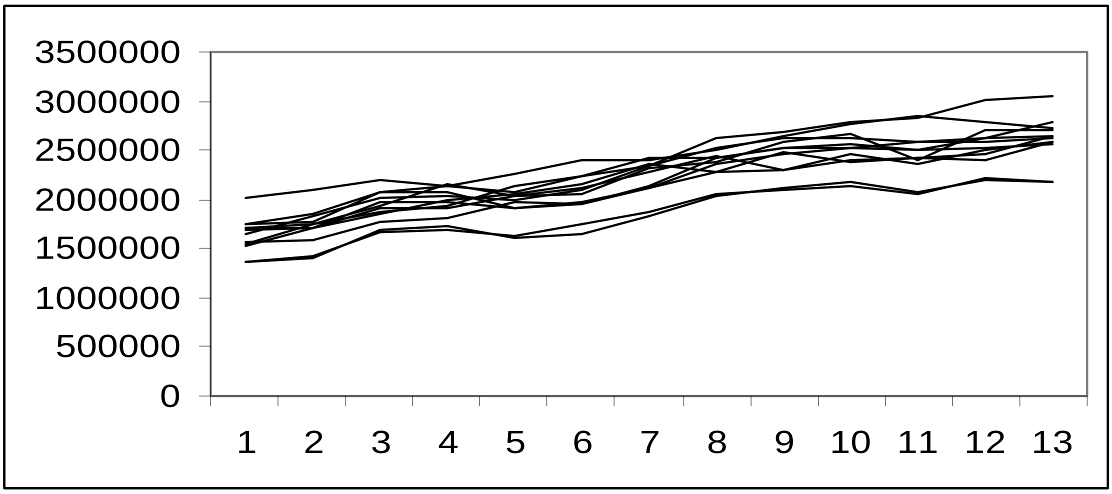


TABLE 1				
Estimation of a deterministic seasonal model				
	White noise		AR(1)	
	An intercept	A linear time trend	An intercept	A linear time trend
$\hat{g}_1$	-6493.5	69164.6*	66049.2**	67115.8**
$\hat{g}_2$	-90292.2	-21512	-25592.8	-22939.1
$\hat{g}_3$	206676.1	268578.2**	263809.1**	267538.7**
$\hat{g}_4$	80189.8	135214**	130024**	134419.5**
$\hat{g}_5$	-327202.2**	-279056**	-284409.1**	-279691.7**
$\hat{g}_6$	-332445.7**	-291177.6**	-296445**	-291705.6**
$\hat{g}_7$	52089.5	86479.5**	81537.9*	86030.5**
$\hat{g}_8$	285765.7**	313277.8**	308893.3**	312894.3**
$\hat{g}_9$	-4189.2	16444.8	12840.9	16125.7
$\hat{g}_{10}$	105069.5	118825.5**	116217.5**	118581.1**
$\hat{g}_{11}$	61154.7	68032.7*	66628.5**	67887.9**
Constant	2181952**	1604198**	2331406**	1605913**
Time trend		6878**		6861.3**
$\hat{r}$			0.965	0.611
Seasonality test	42.9219**	470.94**	963.2331**	855.28**
AIC	-2212.5	-2023.8	-1989.7	-1975.6
SC	-2230.8	-2043.7	-2009.5	-1996.9
$\bar{R}^2$	0.17	0.93	0.94	0.95

The table presents the estimated individual seasonal coefficients. * and ** represents significant at 10% and 5% level. The seasonality statistic tests the null hypothesis of no seasonality. AIC, SC and $\bar{R}^2$ stands for Akaike, Schwartz and adjusted R^2 criteria.

TABLE 2			
Dickey, Hasza and Fuller (DHF, 1984) tests			
	No regressors	An intercept	A linear time trend
White noise	8.32	-3.34**	-2.57
AR (1)	3.94	-2.26	-1.85

** means that we can reject the null hypothesis of seasonal unit root at 5% level.

TABLE 3			
Estimation of d based on seasonally fractionally integrated models ($(1-L^{12})^d$)			
	No regressors	An intercept	A linear time trend
White noise	[0.05 (0.16) 1.10]	[0.15 (0.63) 1.05]	[0.08 (0.32) 1.02]
AR (1)	[0.04 (0.14) 1.11]	[-0.03 (0.18) 0.96]	[-0.07 (0.22) 0.97]

TABLE 4			
Estimation of d based on fractionally integrated models ($(1-L)^d$)			
	No regressors	An intercept	A linear time trend
White noise	[0.49 (0.63) 0.79]	[0.38 (0.43) 0.52]	[0.01 (0.13) 0.34]
Seasonal AR (1)	[0.80 (0.90) 1.03]	[0.52 (0.56) 0.63]	[0.33 (0.43) 0.56]

TABLE 5						
Estimation of d_1 and d_2 based on fractional models at 0 and seasonal frequencies						
	No regressors		An intercept		A linear time trend	
	d_1	d_2	d_1	d_2	d_1	d_2
White noise	0.40 (0.21, 0.59)	0.02 (-0.07,0.21)	0.40 (0.19, 0.59)	0.02 (-0.09, 0.18)	0.13 (0.01, 0.29)	0.02 (-0.08, 0.18)
Seasonal AR (1)	0.39 (0.14, 0.73)	0.03 (-0.10, 0.27)	0.38 (0.13, 0.73)	0.06 (-0.08, 0.20)	0.14 (-0.02, 0.30)	0.04 (-0.11, 0.24)

TABLE 6					
k-ahead forecasting errors according to the selected models					
	DUM	DHF	SFI	FI	FSFI
1	**-0.0753572**	0.5679770	0.4212977	0.7667203	0.4210427
2	**0.0078927**	0.5680356	0.4213547	0.7667912	0.4211010
3	**-0.1607014**	0.6967750	0.5500992	0.8955199	0.5498420
4	**-0.0476954**	0.6835207	0.5368472	0.8822667	0.5365892
5	-0.1658322	0.1581339	0.0114607	0.3568796	**0.0112020**
6	-0.1693482	0.1494642	0.0027920	0.3482093	**0.0025319**
7	**-0.0887552**	0.6146529	0.4679819	0.8133977	0.4677205
8	**0.0812992**	1.0184366	0.8717617	1.2171801	0.8715029
9	**-0.1135775**	0.5336560	0.3869775	0.7323985	0.3867212
10	**0.0165474**	0.7730988	0.6264197	0.9718409	0.6261632
11	**-0.1218879**	0.5908312	0.4441522	0.7895734	0.4438957
12	**-0.0329659**	0.6187227	0.4720475	0.8174660	0.4717882
13	**-0.0612604**	0.5727892	0.5177302	0.8631522	0.5174747
14	**-0.1244034**	0.4264552	0.3713940	0.7168181	0.3711402
15	**-0.0529947**	0.7951982	0.7401415	1.0855624	0.7398852
16	**-0.2474060**	0.4745267	0.4191720	0.7648919	0.4192152
17	-0.1484917	0.1661922	0.1111362	0.4565570	**0.1108802**
18	-0.1696712	0.1398587	0.084804	0.4302228	**0.0845462**
19	**0.0171102**	0.7112355	0.6561822	1.0015994	0.6559232
20	**0.0779644**	1.0058177	0.9507617	1.2961801	0.9505042
21	**-0.0882520**	0.5496975	0.4946382	0.8400586	0.4943819
22	**0.0303140**	0.7775802	0.7225210	1.0679409	0.7222642
23	**-0.0762574**	0.6271780	0.5721177	0.9175388	0.5718619
24	**-0.0748194**	0.5675847	0.5125287	0.8579467	0.5122699

TABLE 7					
Forecasting measures for the selected models					
k = 6	DUM	DHF	SFI	FI	FSFI
MAPE (x1000)	**0.0000427**	0.0001813	0.0001219	0.0002616	0.0001218
MSE	**0.01500**	0.27421	0.15766	0.50079	0.15749
RMPSE	**0.07733**	0.36802	0.26946	0.52371	0.26931
RMSE	**0.12248**	0.52365	0.39707	0.70767	0.39686
MAD	**0.10447**	0.47065	0.32397	0.66939	0.32371
k = 12	DUM	DHF	SFI	FI	FSFI
MAPE (x1000)	**-0.0000300**	0.0002167	0.0001599	0.0002935	0.0001598
MSE	**0.01113**	0.388955	0.24060	0.66004	0.24037
RMPSE	**0.07542**	0.40671	0.32137	0.58731	0.32120
RMSE	**0.10552**	0.62414	0.49051	0.81242	0.49028
MAD	**0.09015**	0.58110	0.43443	0.77985	0.43417
k = 18	DUM	DHF	SFI	FI	FSFI
MAPE (x1000)	**-0.0000381**	0.0001984	0.0001533	0.0002880	0.0001532
MSE	**0.01487**	0.33829	0.22425	0.62979	0.22403
RMPSE	**0.06568**	0.43864	0.33288	0.60124	0.33270
RMSE	**0.12195**	0.58163	0.47355	0.79359	0.47332
MAD	**0.10478**	0.53046	0.41432	0.75974	0.41406
k = 24	DUM	DHF	SFI	FI	FSFI
MAPE (x1000)	**-0.0000305**	0.0002108	0.0001721	0.0003038	0.0001720
MSE	**0.01225**	0.38454	0.28032	0.72682	0.28007
RMPSE	**0.06823**	0.43113	0.35931	0.63093	0.35913
RMSE	**0.11071**	0.62011	0.52945	0.85254	0.52922
MAD	**0.09378**	0.57447	0.47360	0.81903	0.47335

TABLE 8							
Results based on the modified Diebold and Mariano (1995) statistic (M-DM, Harvey et al., 1997) comparing the DUM and the FSFI models							
	h = 1	h = 2	h = 4	h = 8	h = 16	h = 20	h = 22
Absolute P.E.	**-6.481** **(0.000)**	**-5.778** **(0.000)**	**-8.724** **(0.000)**	**-11.124** **(0.000)**	**-2.892** **(0.020)**	-1.4029 (0.233)	-3.455 (0.075)
Squared P.E.	**-5.518** **(0.000)**	**-5.016** **(0.000)**	**-7.362** **(0.000)**	**-5.497** **(0.000)**	**-6.686** **(0.000)**	**-4.994** **(0.008)**	-4.104 (0.055)

p-values in parentheses.